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Section 1

OUTPUT POWER FROM A SELF-EXCITED ELECTROSTATIC GENERATOR

by

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Introduction

Interest in the generator whose basic circuit will be discussed stemmed from the desire for a high voltage generator of compact design for laboratory use. There was the further desire to provide self-excitation so that the maximum voltage attainable would be determined by the control circuitry rather than by an external voltage source or by number of multiplier stages. The practical voltage limitation for such a device is, of course, the breakdown limit of the components and thus requires additional cascaded components for still higher voltages.

The resulting design was a variable capacity generator, many of whose properties and design limitations, as the author soon discovered, were similar to those of other designs.[1-3] With the type of excitation provided in this design it is of interest to compute output power and other quantities for comparison with other designs.

The Basic Circuit

Figure 1 shows a self-excited generator with positive output voltage. Negative polarity can be obtained by grounding point 0 instead of G or by reversing directions of all the diodes. During any one cycle of operation, point 0 increases in potential as capacitance C is reduced. With reduction of C by a factor greater than two, capacitors C_1 and C_0 are charged in series. Point 0 subsequently drops in potential as C increases so that C_1 and C_0 discharge in parallel into C. They thus return more charge to C than they take from C each cycle. As a result, voltages on all condensers increase with time unless a sufficiently large load drains off the excess charge accumulation.

A small priming charge is or is not needed to start the generation of voltage depending on the size of thermal or other induced voltages appearing in the circuit.

Properties of the Generator Action

The dc voltage across the variable capacitor, C, has a large ac component. The harmonic content of this component is determined by the way in which C varies with time, which in turn is affected by the geometry of C. Waveforms at certain points in the circuit are demonstrated in Fig. 2,

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for small and large values of C_0 and C_1. One notes the exponential growth of voltage with time for fixed load and generator parameters until voltage breakdown across C occurs. Shown is one example where a nonlinear load resistance arrests the voltage growth. Also shown is the effect of a quenching resistance which retains most of the stored energy of the system during sparking of the variable capacitor.

For fixed parameters, the growth of the voltage envelope has the form

$$V = V_o e^{t f \ln(1+F)} \tag{1}$$

where t is time and f is condenser frequency. The fractional voltage increase per cycle, F, can be shown to be

$$F = \frac{\Delta V}{V} = \frac{C_S C_M (1-2r)(2\lambda - 1)}{(C_s + \lambda C_m)(C_M + C_1 + C_0)} \tag{2}$$

where the symbols have the following meanings:

C_s = series capacitance of C_0 and C_1

C_M = maximum capacitance of C

C_m = minimum capacitance of C

$r = \dfrac{C_m}{C_M}$

$\lambda = 1 - \frac{1}{2}\frac{Z_e}{Z}$ = load parameter (equals $\frac{1}{2}$ for equilibrium voltage).

The load impedance for equilibrium voltage is

$$Z_e = \frac{4 C_s + C_M}{C_s C_M f(1-2r)}. \tag{3}$$

If impedance $Z > Z_e$, voltage increases with time. If $Z < Z_e$, the voltage decreases with time.

For equilibrium voltage operation, clearly some kind of regulation is required. Automatic regulation can be built around a controllable parasitic load or some of the quantities affecting Z_e such as C_s or stray capacitances, which are not shown in the above formula. The capacity ratio r (and C_M) can be effectively varied by adding a controllable capacitance across C.

It is clear from the expression for F that no generator action is sustained if $r > \frac{1}{2}$. A circuit which has self-excitation for $r > \frac{1}{2}$ will be described later.

Maximum Power Considerations

Power delivery to the load under equilibrium conditions is given by

$$P \approx \frac{(4\ C_s + C_M)\, C_s\, C_M\, (1 - 2r)\, f}{(2\ C_s + C_m)^2}\, \nu^2 \approx \frac{C_s\, C_M\, (1 - 2\ r)\, f}{4\ C_s + C_M}\, V^2 \qquad (4)$$

where ν is the minimum voltage appearing across the rotary condenser gap and V the maximum, or output, voltage. With a fixed rotary condenser geometry, one is free to vary the external circuit components, namely C_s, in an attempt to maximize the power. Although the condition for maximum rate of voltage build-up is $C_0 \approx C_1 \approx 2\ C_s \approx \sqrt{C_m\, C_M}$, this does not yield maximum power.

Setting the derivative of P with respect to C_s equal to zero yields

$$C_s \approx \frac{1}{2}\, \frac{r\, C_M}{1 - 4r}\, , \quad \text{for } r < \frac{1}{4} \qquad (5)$$

$$P \approx \frac{1}{4}\, C_M\, f\, \nu V \approx \frac{1}{2}\, C_M\, r\, f\, V^2 \approx \frac{1}{8\, r}\, C_M\, f\, \nu^2 \qquad (6)$$

$$V \approx \frac{\nu}{2\, r}\, . \qquad (7)$$

The maximum power is thus obtained only if the voltage breakdown limit is reached during the low voltage (maximum capacity) part of the cycle. Whether or not this is true depends on condenser geometry. One might expect the breakdown limit to occur more probably at high voltage. In this event and in any case for $r > \frac{1}{4}$, one should make $C_s/C_M >> 1$ for maximum power, giving

$$P \approx \frac{1}{4}\, C_M\, f\, (1 - 2\ r)\, V^2 \approx C_M\, (1 - 2\ r)\, f\, \nu^2 \qquad (8)$$

$$V \approx 2\ \nu \qquad (9)$$

$$Z_e \approx 4/C_M\, f\, (1 - 2\ r)\, . \qquad (10)$$

It is interesting to note that expression (8) is very nearly the same as the power for generator types III and IIIa discussed by Denholm et al.[3] when r is not too close to the value $\frac{1}{2}$.

It remains to consider the geometry of the rotary condenser which is desired for maximum power. It is apparent that high capacity variation frequency, f, is desirable, where $f = n\,p$ is the product of revolution frequency and number of ridges or blades on the rotor or stator. One makes p large by making the blade spacing as small as feasible at a sacrifice of increase in r for a fixed condenser gap thickness. Also the maximum capacity increases as the ratio of blade separation to blade width is reduced. Other investigators[3,4] have examined the problem of rotor geometry in some detail, especially the effect of blade dimensions on capacity, mechanical instability problems, and power to mass ratio. They have taken a blade width to spacing ratio equal to one for most calculations. Whether or not this is the optimum ratio depends on blade profile, voltage change during a cycle, the value of r, in short the interaction of all parameters appearing in expression (8). To the best of the author's knowledge, this problem is not yet solved. It is further complicated by the fact that voltage breakdown in vacuum is not at present sufficiently well understood for precise prediction or control to be made. We have tacitly assumed that vacuum is the insulating medium which is a natural choice for space application and does not reduce generator efficiency as would pressurized gas.

For vacuum insulation and for fixed relationships between blade and gap dimensions, one can vary the minimum gap thickness d and see how the output power varies. Under such conditions, r remains constant as does the capacitance per blade.[3] If one assumes the breakdown voltage to follow the approximate empirical relation, $V \approx k\,d^{1/2}$, the number of blades will increase as $1/d$ when d decreases so that one has the following variations with d:

$$C_M \propto \frac{1}{d}; \quad f \propto \frac{1}{d}; \quad V^2 \propto d.$$

Thus the power increases as $1/d$. The minimum d is set by practical mechanical tolerance limits.

There is a premium, therefore, on close tolerances. If the voltage becomes lower than desired for small d and maximum power, condenser units can be stacked and retain the self-excitation feature of Fig. 1. If one stage has maximum voltage V and power P, the output voltage of N stages will be N V at power N P. Such a circuit is given in Fig. 3.

Other mechanical problems such as bearings and seals will not be treated here. They are the subject of intensive investigation in many places. The assumption is made that these problems are solvable for high speed, high vacuum operation.

Self-Excitation for $r > \frac{1}{2}$

If the capacity ratio of the rotary condenser is greater than $\frac{1}{2}$, the generator of Fig. 1 does not work. There are a number of modifications one can devise which will work. For example, if all three condensers C, C_0, and C_1 are made variable with C_0 and C_1 operated out of phase with C as in Fig. 4, the capacity ratios must satisfy the relation, $r (r_0 + r_1) < 1$. If all ratios are chosen equal, then $r < 1/\sqrt{2}$ is required. An extension of this scheme is the symmetrical design shown in Fig. 5, where C_0 and C_1 are operated out of phase with C_2 and C_3. With four identical capacitors, $r < 1/2$ is required.

Two other schemes using voltage multiplication are shown in Figs. 6 and 7. They can in principle be used with r approaching the value one if a sufficiently large number of multiplying stages are used. They have in common the drawback of requiring a commutator.

In Fig. 6 a Greinacher type of voltage multiplication is used which acts on the ac component of the rotary condenser output voltage. The total circuit output voltage must be raised to a value greater than twice the minimum condenser voltage in order to charge C_0 and C_1 in series. The number of multiplying stages required is $N > (2r - 1)(1 - r)$ with the assumption of ideal multiplying stages. This voltage multiplying feature, of course, can also be used to obtain higher voltages than a single disk generator with $r < \frac{1}{2}$ gives by building it atop the basic circuit of Fig. 1.

Another voltage multiplying scheme is that of cascaded generators[3] shown in Fig. 7 to give self-excitation for any r-value. With identical generator stages, the number of stages required is $N > r/1 - r$.

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3. A. S. Denholm et al., "Feasibility and Design Study for Electrostatic Generators", WADD Technical Report 61-105, July, 1961.

4. C. Coenraads et al., "Electrostatic Power Generators for Space", ARS Space Power Systems Conference Paper No. 2555-62, September, 1962.

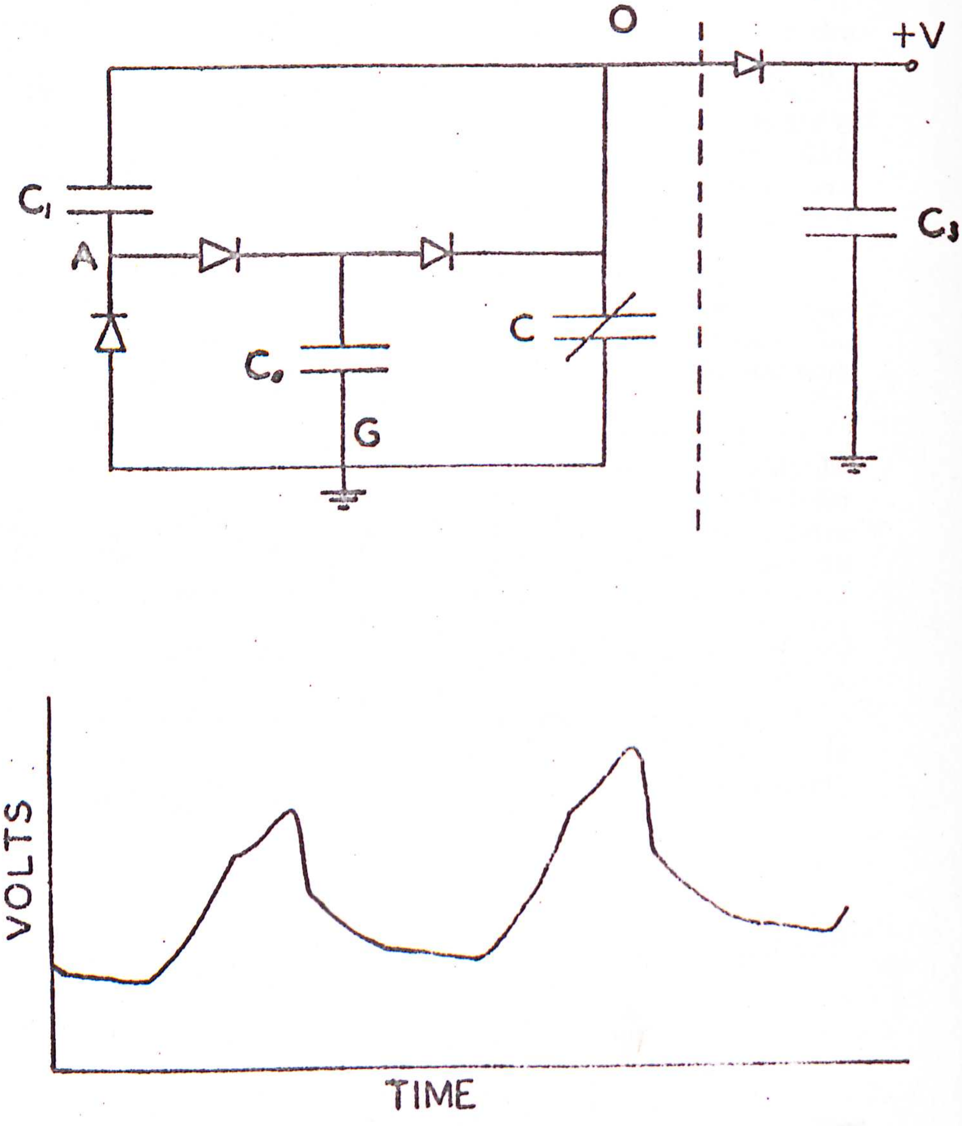
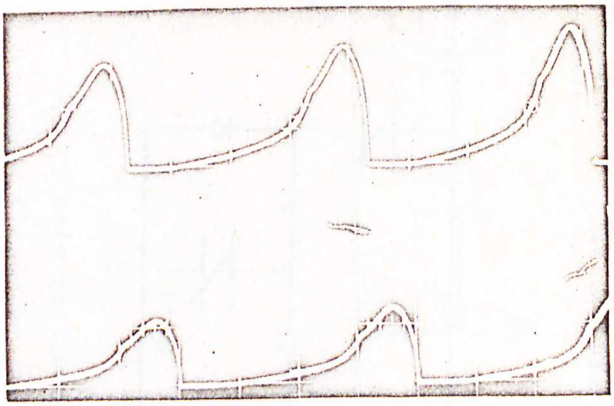
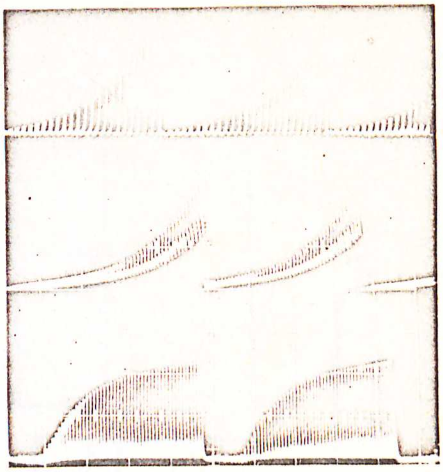


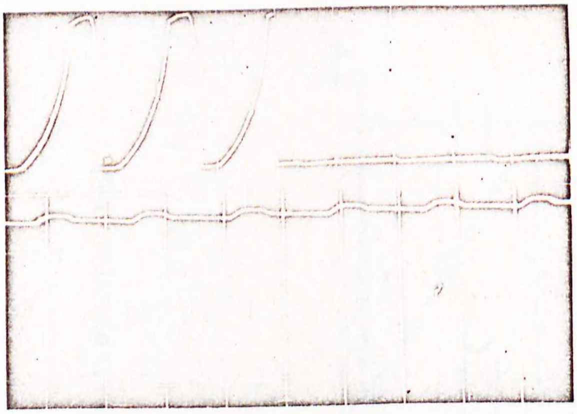
Fig. 1. Basic generator. Voltage waveform is for point O.



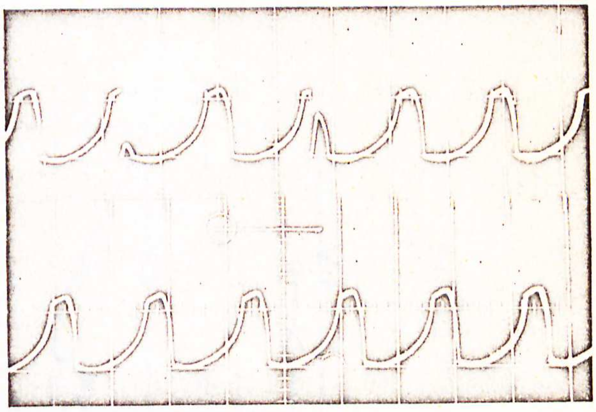
a. Voltage at point A during buildup with condensers C_0 and C_1 small.



b. Exponential growth and breakdown. 1) Voltage at point A. 2) Voltage across C_0. 3) Effect of nonlinear load resistance on growth rate.



c. Voltages with C_0 and C_1 much larger than C_M. Voltage at point A showing breakdown and voltage across C_0.



d. Voltage across C with a quenching resistance in series with C. Fast recovery is shown following occasional breakdown.

Fig. 2. Waveforms for a test generator. Frequency is 60 c. p. s.

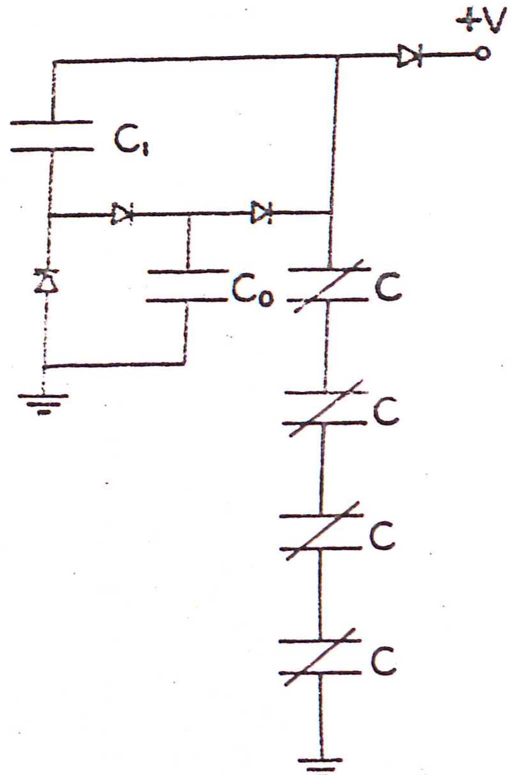


Fig. 3. Cascade generator with $r<½$

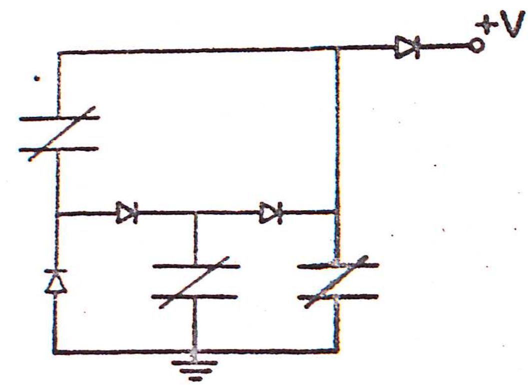


Fig. 4. Generator with $r<1/\sqrt{2}$

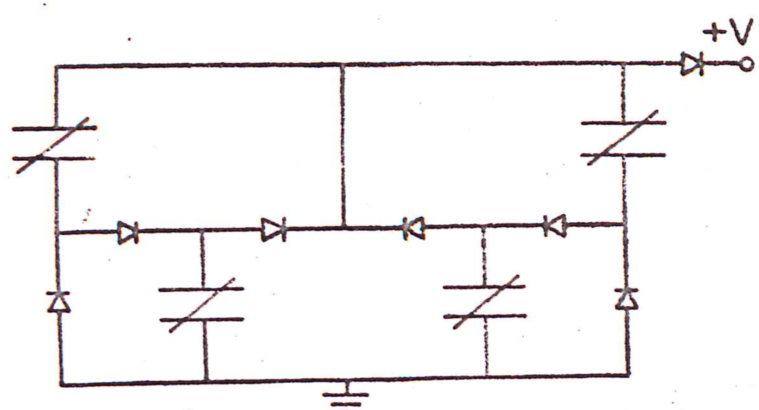


Fig. 5. Balanced generator with $r<1/2$

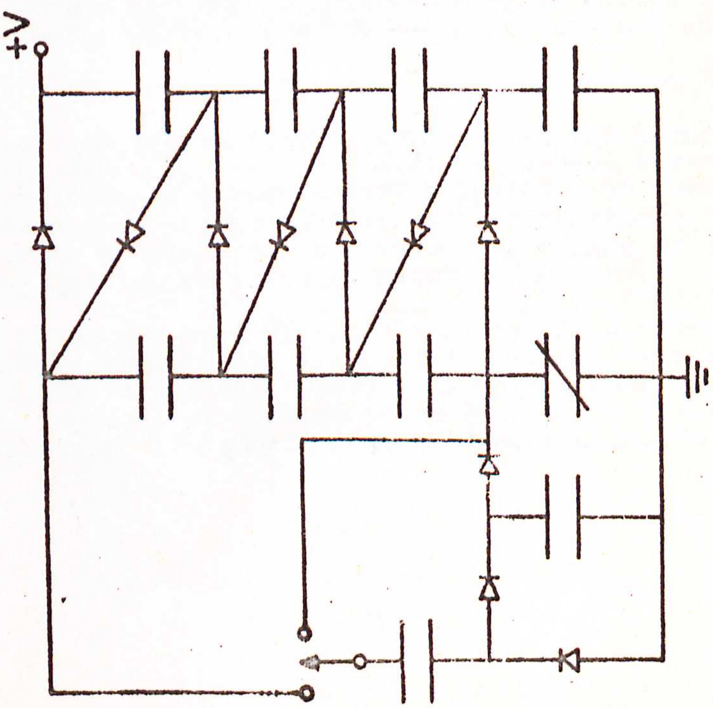


Fig. 6. Generator with r>½.

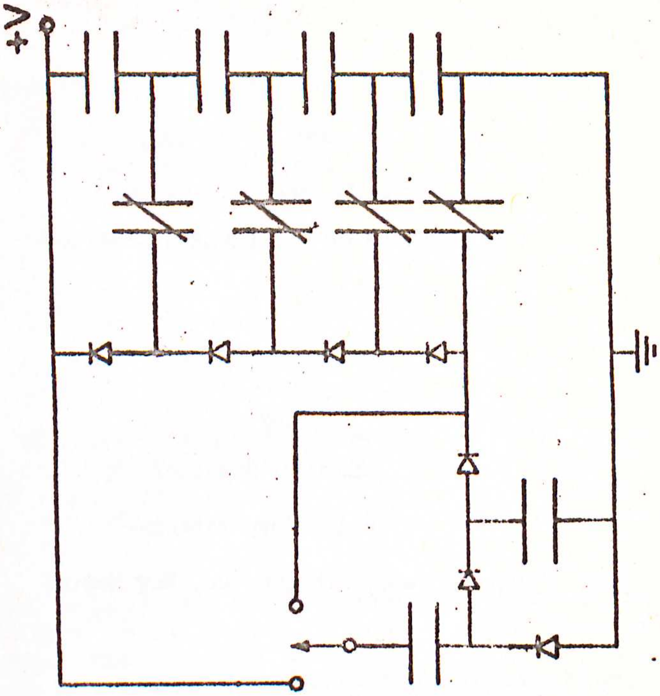


Fig. 7. Cascaded generator with r>½.